

1.1. Linear Equations in One Variable

* Definition (Def) :

1. An equation is a statement that indicates that two quantities are equal.

Ex: $x = -4$
 $p + 3 = 11$

2. A solution to an equation is a value of the variable that makes the equation a true statement.

3. A linear equation in one variable is an equation that can be written in the form:

$$ax + b = 0 \quad \left\{ \begin{array}{l} x: \text{variable} \\ a, b: \text{real number} \\ (a \neq 0) \end{array} \right.$$

Ex: $4x - 3 = 0$ (linear)

$$\frac{4}{5}p - \frac{3}{10} = 0 \text{ (linear)}$$

$$4x^2 + 8 = 0 \text{ (Not linear)}$$

⊗ Property of Equality:

1. Addition: If $a = b$, then $a + c = b + c$ (a, b, c : real numbers)

2. Subtraction: If $a = b$, then $a - c = b - c$

3. Multiplication: If $a = b$, then $a \cdot c = b \cdot c$

4. Division: If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ ($c \neq 0$)

Notice: Subtraction / Division Property follow directly from addition / multiplication

→ $a - c = a + (-c)$

$b - c = b + (-c)$

⇒ If $a + (-c) = b + (-c)$,

then $a - c = b - c$

→ $\frac{a}{c} = a \cdot \frac{1}{c}$

⇒ If $a \cdot \frac{1}{c} = b \cdot \frac{1}{c}$,

$\frac{b}{c} = b \cdot \frac{1}{c}$

then $\frac{a}{c} = \frac{b}{c}$ ($c \neq 0$)

⊕ Solving Linear Equation :

Step 1 : Simplify both sides of the equation

- Clear parentheses
- Clear fractions / decimals

Step 2 : Isolate the variable

- Applying Property of Equality

Step 3 : Solve and check the solution

- Write the solution set

Ex: $3x + 1 = -7$

Step 1: $3x + 1 = -7$
 -1 -1

Step 2: $\frac{3x}{3} = \frac{-8}{3}$

Step 3: $x = -\frac{8}{3}$

Check: $3(-\frac{8}{3}) + 1 = -8 + 1 = -7$ ✓

Notice : ⇒ Solution is presented by $\{ \}$, Ex: $\{8\}$

Solution set: $\{-\frac{8}{3}\}$

⇒ An equation can have

1 solution : Ex: $\{8\}$

No solution : Ex: \emptyset - empty set

Many solutions : Ex: All real numbers

Ex: a) $x + 4 = 6$

b) $x + 1 = x + 2$

c) $2x + 4 = 2(x + 2)$

Ex 1: Solving a linear Equation:

a) $2x - 5 = -11$

b) $5x - 19 = -23$

c) $11z + 2 = 5(z - 2)$

d) $-3(x+4) + 2 = 7 - (x+1)$

e) $-4[y - 3(y-5)] = 2(6 - 5y)$

* Case: Solving linear equation by clearing fraction/decimal

Fraction:

Step 1: Find LCD (least common denominator)
then multiply both sides of the equation
by LCD.

Step 2: Simplify both sides of the equation
• Distributive property

Step 3: Isolate the variable

Step 4: Check the solution

Ex: $\frac{1}{4}w + \frac{1}{3}w - 1 = \frac{1}{2}w - 2$

Notice: LCD, 12

Step 1: $12 \cdot \left(\frac{1}{4}w + \frac{1}{3}w - 1 \right) = 12 \cdot \left(\frac{1}{2}w - 2 \right)$

Step 2: $12 \left(\frac{1}{4}w \right) + 12 \left(\frac{1}{3}w \right) - 12(1) = 12 \left(\frac{1}{2}w \right) - 12(2)$

$$3w + 4w - 12 = 6w - 24$$

$$7w - 12 = 6w - 24$$

Step 3: $\begin{array}{r} 7w - 12 = 6w - 24 \\ +12 \qquad +12 \end{array}$

$$7w \qquad = 6w - 12$$

$$-6w \qquad -6w$$

$$w = -12$$

Step 4: $\frac{1}{4}(-12) + \frac{1}{3}(-12) - 1 = -3 + (-4) - 1 = -8$

$$\frac{1}{2}(-12) - 2 = -6 - 2 = -8 \quad \checkmark$$

Decimal:

Step 1: Rewrite decimal as fraction

Step 2: Follow the process "solving linear equation by clearing fraction" above.

Ex: $0.55x - 0.6 = 2.05x$

Ex 2: Solving a linear equation:

a) $\frac{1}{8} - \frac{x+3}{4} = \frac{3x-2}{2}$

b) $2.2x + 0.5 = 1.6x + 0.2$

HW: 1.1 page 54-56 : # 30, 42, 88